Resit for Final Examination

Name:

Student number:

E-mail address:

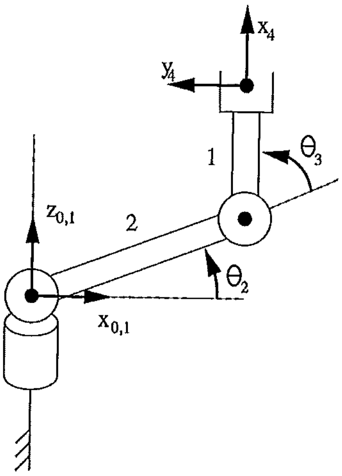
1. Print your name and student number in the space above clearly.
2. Only the textbook is allowed to be used in the exam. You should NOT use your homework, lecture notes, etc. as references.
3. Write your answers on this handout; if necessary, attach extra pages for scratch work.
4. Difficult questions are not necessarily worth more. Therefore, be sure not to spend too much time on any one question. If you have trouble on a question come back to it later.
5. **Please write in English as legibly as possible** – I can't grade what I can't read!
6. If a question is unclear, make an appropriate assumption that does not contradict any information given in the question.
7. This exam must be completed within the assigned time.
8. There are altogether 6 problems, 100 points.

Problem 1: (15 points) (a)Given the following transformation matrix ${}^B_AT$, please find ${}^A_BT$:

$$
{}^B_AT = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & \cos\theta & -\sin\theta & 2 \\ 0 & \sin\theta & \cos\theta & 3 \\ 0 & 0 & 0 & 1 \end{bmatrix}.
$$

(b) Given $\theta = 45°$ and ${}^BP = [4\ 5\ 6]^T$, compute ${}^AP$.

Problem 2: (30 points) Consider the RRR manipulator shown below.



(a) Assuming that the joint limits of this manipulator are $0 \leq \theta_2 \leq 180°$ and $-90° \leq \theta_3 \leq 90°$, sketch the workspace of this manipulator.
(b) Find the DH parameters for this manipulator in the form of a table.
(c) Find the basic Jacobian, J_0, for this manipulator.
(d) Find ${}^1J_v$, the Jacobian matrix expressed in frame $\{1\}$.
(e) Use the matrix that you found in (d) to find the singularities (with respect to linear velocity) of this manipulator.

Problem 3: (15 points) General mechanisms sometimes have certain configurations, called "isotropic points," where the columns of the Jacobian become orthogonal and of equal magnitude. If a manipulator is at an isotropic point and its corresponding Jacobian can be written in the following form with missing elements, please determine the missing elements.

$$\begin{bmatrix} ? & 0 & ? \\ 0.707 & ? & ? \\ ? & ? & 0 \end{bmatrix}$$

Problem 4: (15 points) Consider a RRRR manipulator, whose forward kinematics are

$$
{}^0_4T = \begin{bmatrix} c_{12}c_{34} - \frac{\sqrt{2}}{2}s_{12}s_{34} & -c_{12}s_{34} - \frac{\sqrt{2}}{2}s_{12}c_{34} & \frac{\sqrt{2}}{2}s_{12} & \sqrt{2}c_{12}c_3 - s_{12}(s_3 - 1) + c_1 \\ s_{12}c_{34} + \frac{\sqrt{2}}{2}c_{12}s_{34} & -s_{12}s_{34} + \frac{\sqrt{2}}{2}c_{12}c_{34} & \frac{\sqrt{2}}{2}c_{12} & \sqrt{2}s_{12}c_3 + c_{12}(s_3 - 1) + s_1 \\ \frac{\sqrt{2}}{2}s_{34} & \frac{\sqrt{2}}{2}c_{34} & \frac{\sqrt{2}}{2} & s_3 + 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

and rotational Jacobian

$$
{}^0J_\omega = \begin{bmatrix} 0 & 0 & \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \\ 0 & 0 & 0 & 0 \\ 1 & 1 & \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \end{bmatrix}.
$$

A general force vector is applied to the origin of frame $\{4\}$ and measured in frame $\{4\}$ to be $[0, 6, 0, 7, 0, 8]^T$. If the joints variables are $[0, 90^\circ, -90^\circ, 0]^T$, please determine the joint torques that statically balance the general force.

Problem 5: (10 points) We want to design a humanoid robotic arm, i.e. a robotic arm that functions like a human arm.
(a) How many degrees of freedom do you think the robotic arm should have?
(b) How many revolute joints and prismatic joints will you use in the design?
(c) Draw an illustrative diagram to show how you want to connect the chosen joints together by links and please attach frames to the joints together with the base and the end-effector. (You don't need to spend a lot of time on making your drawing "beautiful". It suffices as long as it is clear.)

Problem 6. (15 points) Consider a manipulator for which the equation of motion is

$$\dot{x}_1 = x_2$$

$$\dot{x}_2 = -a \sin x_1 - b x_2$$

(a) Is this system linear or nonlinear?
(b) Show that $(0,0)$ is an equilibrium of the system.
(c) Show that $V(x) = a(1 - \cos x_1) + \frac{1}{2}x_2^2$ can be used as a Lyapunov function. Use it to prove the stability of the system at the origin.